Curtin University, Semester 2, 2026

ECOM5005

Tutorial 6

Sampling Distributions

Preparation expectation. Remember that you are expected to prepare for the tutorial prior to your tutorial. In particular, if you cannot obtain answers then you should demonstrate that you have attempted them by providing your own questions concerning these problems, the relevant section(s) of the lecture slides, etc.

Learning Outcomes

1. Interpret the concept of the sampling distribution.
2. Calculate probabilities related to the sample mean and the sample proportion.
3. Recognise the importance of the Central Limit Theorem.

PART 1 - Short answer questions

Question 1. Means in quality control

An auto-maker does quality control tests on the paint thickness at different points on its car parts since there is some variability in the painting process. A certain part has a target thickness of 2mm. The distribution of thickness on this part is skewed to the right with a mean of 2mm and a standard deviation of 0.5mm. A quality control check on this part involves taking a random sample of 100 points and calculating the mean thickness of those points. Assuming the stated mean and standard deviation of the thickness are correct, what is the probability that the mean thickness in the sample of 100 points is within 0.1mm of the target value?

Part 1: Establish normality

(1) What is the shape of the sampling distribution of the sample mean thickness? A. Skewed to the right B. Approximately normal C. Not enough information

Part 2: Find the mean and standard deviation of the sampling distribution

(2) What is the mean of the sampling distribution of $\bar{x}$?

(3) What is the standard deviation of the sampling distribution of $\bar{x}$?

Part 3: Use normal calculations to find the probability in question

(4) What is the probability that the mean thickness in the sample of 100 points is within 0.1mm of the target value? A. 68% B. 90% C. 95% D. 99.7%

Question 2. Proportions in polling results

According to the US Census Bureau's American Community Survey, 87% of Americans over the age of 25 have earned a high school diploma. Suppose we are going to take a random sample of 200 Americans in this age group and calculate what proportion of the sample has a high school diploma. What is the probability that the proportion of people in the sample with a high school diploma is less than 85%?

Part 1: Establish normality
(1) What is the expected number of people in the sample with a high school diploma?
(2) Is the sampling distribution of $\hat{p}$ approximately normal?

Part 2: Find the mean and standard deviation of the sampling distribution
(3) What is the mean of the sampling distribution of $\hat{p}$?
(4) What is the standard deviation of the sampling distribution of $\hat{p}$? (You may round your answer to three decimal places.)

Part 3: Use normal calculations to find the probability in question
(5) What is the probability that the proportion of people in the sample with a high school diploma is less than 85%? A. ≈0.17 B. ≈0.20 C. ≈0.23 D. ≈0.26

Extension problems - real-life applications

Extension Problem A - Airline turnaround-time reliability

An airline's aircraft turnaround time at a major airport is strongly right-skewed, with population mean $\mu = 42$ minutes and population standard deviation $\sigma = 15$ minutes. Operations managers review a random sample of 64 flights each week and use the sample mean turnaround time as a KPI.

(a) Explain why the sampling distribution of $\bar{x}$ can be treated as approximately normal.

(b) Find $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$.

(c) Find $P(\bar{x} > 45)$. Interpret this probability as an operational risk.

(d) Find $P(40 < \bar{x} < 44)$.

Extension Problem B - Streaming-platform conversion-rate monitoring

A streaming platform knows that 36% of free-trial users convert to a paid subscription. Each week, the analytics team randomly samples 400 trial users and calculates the sample conversion proportion $\hat{p}$.

(a) Check whether the sampling distribution of $\hat{p}$ is approximately normal.

(b) Find the mean and standard error of $\hat{p}$.

(c) Find $P(\hat{p} < 0.33)$.

(d) Find $P(0.34 < \hat{p} < 0.39)$ and interpret the result.

Extension Problem C - Hotel check-in times and the effect of sample size

Check-in times at a large hotel are strongly right-skewed with mean $\mu = 7.5$ minutes and standard deviation $\sigma = 4.8$ minutes. Two managers independently monitor service performance: Manager A uses samples of $n = 16$ guests, while Manager B uses samples of $n = 64$ guests.

(a) Which manager can more safely use a normal approximation for the sampling distribution of $\bar{x}$, and why?

(b) Calculate the standard error for each manager.

(c) For Manager B, find $P(6.5 < \bar{x} < 8.5)$.

(d) Explain how increasing the sample size changes the precision of the sample mean.

PART 2 - Data exercise

Question 3. Restaurant waiting-time sampling experiment

Download the data file titled "Restaurant_Times.xlsx" from the data section in Topic 6. Open the file in Excel. The data contain 100 restaurants and three variables: Waiting Time, Serving Time, and Complimentary Drink (Yes = 1, No = 0). Complete the following tasks.

(a) Use Excel's Data Analysis > Descriptive Statistics tool to calculate descriptive statistics for all three variables. Interpret the mean of the binary Complimentary Drink variable.

(b) Create a histogram (or a column-chart approximation to a histogram) for Waiting Time. Describe the shape of the distribution.

(c) Generate seven random samples of 30 waiting times. For this classroom simulation, sample with replacement. Freeze each sample by Copy > Paste Special > Values, then calculate its sample mean.

(d) Compare each of the seven sample means with the overall mean waiting time for all 100 restaurants.

(e) Calculate the mean of the seven sample means and the standard deviation of the seven sample means. How do these compare with the overall mean and the expected sampling variability?

(f) Estimate the standard error of the mean for samples of size 30 using $s/\sqrt{30}$, where s is the standard deviation of all 100 waiting times. Compare it with the observed standard deviation of the seven sample means.

(g) Create a chart showing the seven sample means together with the overall mean. Use your results to explain the Central Limit Theorem and why sample means vary less than individual waiting times.

PART 3 - Pearson MyLab

1. Complete the Homework #3 questions on MyLab.

Note: If you are having issues with obtaining access to MyLab, please re-read the instructions (available in the welcome topic). If you are still unsuccessful, please send an email to the teaching team explaining the issue. It is important that you complete MyLab tasks on a weekly basis since the final e-test in this unit will be completed using MyLab.